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	D4.1 – SINGLE-MODE OPTICAL VERIFICATION Version 0.3 – Final PUBLIC
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Single-mode optical verification

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ACRONYMS

DHD	Double Homodyne Detection
HWP	Half-Wave Plate
PBS	Polarising Beam Splitter
PCA	Principal Component Analysis
POVM	Positive-Operator-Valued Measure
SFG	Sum-Frequency Generation
SHG	Second-Harmonic Generation
SPOPO	Synchronously Pumped Optical Parametric Oscillator
WP	Work Package

PUBLISHABLE SUMMARY

This report outlines an advanced certification protocol aimed at verifying key quantum features, such as Wigner negativity and stellar rank, which are essential for achieving quantum computational advantage. This protocol involves calculating the fidelity between an experimentally generated state and a target state, providing a measure of how closely the experimental state matches the ideal target.

The experimental implementation of this protocol employs a technique called double-homodyne detection (DHD). This method simultaneously measures two quadratures of a quantum state, effectively squeezing the state to reveal its characteristics. The DHD's ability to tune the squeezing angle allows researchers to observe transitions between different quantum states.

One of the main uses of this protocol is to verify the generation of a non-Gaussian state, specifically a photon-subtracted squeezed state. This is achieved using a femto-second laser system to produce the necessary quantum states, followed by a mode-selective photon subtraction process. Despite high-quality initial conditions, the current setup only achieved a 31% fidelity in producing the desired non-Gaussian state, indicating a need for further refinement in the photon subtraction process.

1| PROTOCOL

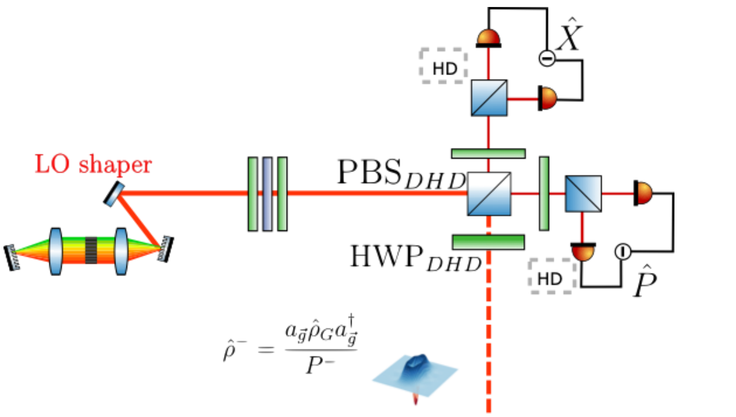
The certification protocol enables the verification of quantum features such as Wigner negativity and stellar rank, establishing a benchmark and a confidence interval for each property [1]. These two quantum features are required for achieving a quantum computational advantage [2,3]. The protocol consists on calculating the "distance" from the generated state to a wisely chosen target state via the fidelity given by

$$F(\hat{\rho}_{\mathrm{exp}}, \Psi_{\mathrm{target}}) = \int P_{\mathrm{target}}(\alpha)\, Q_{\mathrm{exp}}(\alpha) d\alpha,$$

where P_{target} is the Glauber-Sudarshan function (or some regularisation thereof) of the target state, and Q_{exp} is the Husimi function of the state under certification. At the end of the protocol, the fidelity estimated from measurement samples will indicate the probability of achieving the ideal fidelity within the confidence interval, provided the benchmark that witnesses quantum features is surpassed.

2| DOUBLE-HOMODYNE DETECTION

The certification protocol can be implemented experimentally via the double-homodyne detection (DHD, sometimes also referred to as "heterodyne" detection) (fig. 1). The scheme consists of a mode-selective measurement that permits the simultaneous noisy measurement of the quadratures of the quantum state.



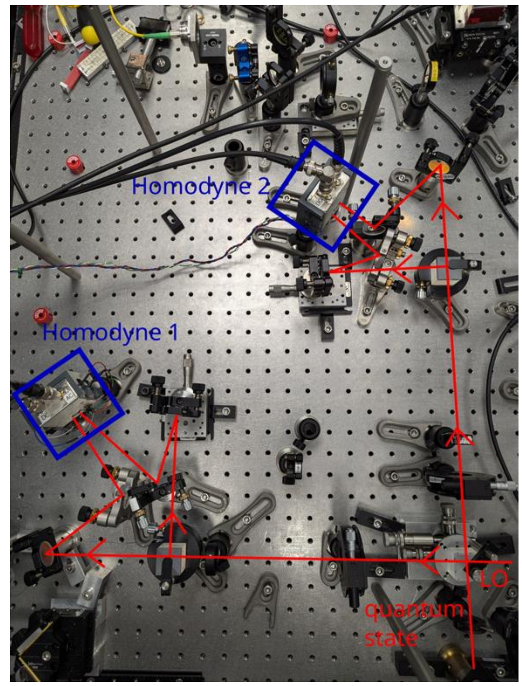


FIGURE 6 SCHEME AND IMAGE OF THE EXPERIMENTAL SETUP OF THE DOUBLE HOMODYNE DETECTOR

The Positive Operator-Valued Measure (POVM) of the DHD is given by:

$$\hat{\prod}_{DHD} = \frac{1}{\pi}\hat{S}(\zeta)\hat{D}(\alpha)|0\rangle\langle 0|\hat{D}^{\dagger}(\alpha)\hat{S}^{\dagger}(\zeta),$$

with $\alpha = (q/r + ip/t)/(2\sigma_0)$ and $\zeta = ln(r/t)$. Here q and p correspond to the phase space coordinates, r and t corresponds to the coefficient of reflection and transmission of PBS_{DHD}, which is controlled by HWP_{DHD} and σ_0 the confidence interval. Therefore, the probability of detecting the state $\hat{\rho}_{exp}$ at a given α is:

$$P_{\mathrm{DHD}}(\alpha) = Q_{\hat{S}(-\zeta)\hat{\rho}_{exp}\hat{S}^{\dagger}(-\zeta)}(\alpha)$$

This equation indicates that according to the values of r/t at PBS_{DHD}, the double-homodyne detection acts as an effective squeezer on the quantum state. This can be seen in figure 2 where by tuning the angle of HWP_{DHD} we observe the transition from a $\hat{X}$-quadrature-squeezed state, to an effectively $\hat{P}$-quadrature-squeezed state.

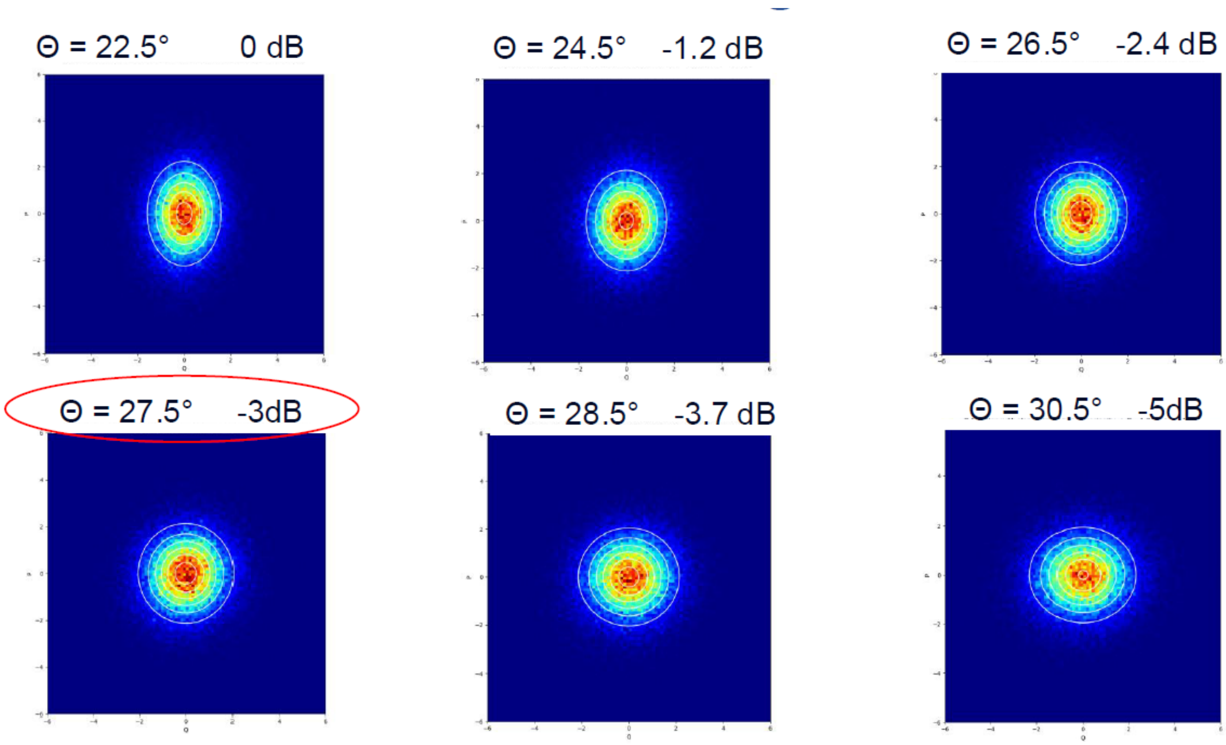


FIGURE 7 ILLUSTRATION OF THE EFFECT OF UNBALANCING ON A GAUSSIAN SQUEEZED STATE WITH 3DB OF SQUEEZING

3| VERIFICATION OF A NON-GAUSSIAN STATE

The state that we aim to certify is a photon-subtracted squeezed state. Figure 3 represents the experimental scheme for its generation. We work with a femto-second laser with a repetition rate of 76 MHz centred at 795 nm. After a second-harmonic generation (SHG), the pump, centerer at 397.5 nm, is sent to a Synchronously Pumped Optical Parametric Oscillator (SPOPO), which generates the set of multimode quantum states. Subsequently, a mode-selective photon subtraction is implemented via a non-linear interaction between a gate pulse beam and the multimode state. A photon created by the sum-frequency generation (SFG) will herald the non-Gaussian state generation in a specific mode.

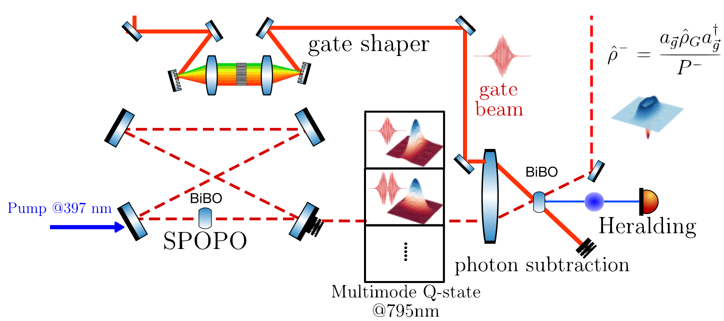


FIGURE 8 EXPERIMENTAL SCHEME FOR MODE-SELECTIVE PHOTON SUBTRACTION

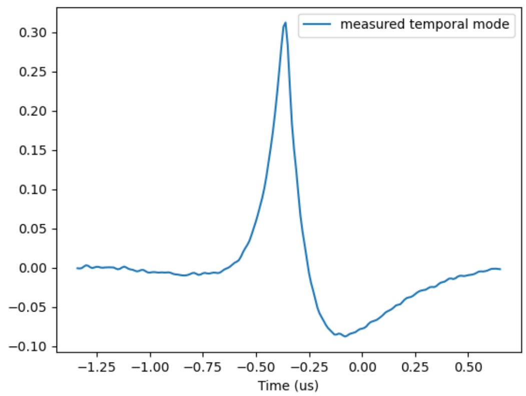


FIGURE 9 RECONSTRUCTED TIME MODE

We see the signature of the single photon subtracted state by performing the Principal Component Analysis (PCA) on our data, which exhibits the temporal mode of our state shown in figure 4. The theoretical mode is a double decaying exponential, but it has been deformed due to the electronics of the system. By applying this temporal mode on our data, we can reconstruct the Q function of the non-Gaussian state, which is shown in figure 5.

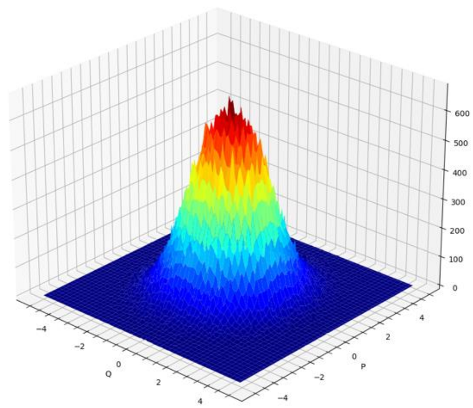


FIGURE 10 SAMPLED Q FUNCTION FOR PHOTON-SUBTRACTED STATE

In fig 2, we showed that we can reproduce the experimental Q-function of a squeezed state, now let us try to reproduce that of a non-Gaussian state. We took 10^6 samples of quadratures from a photon-subtracted squeezed state. The initial squeezed state has 3dB of squeezing with 86% purity. We subtract a photon, then perform double homodyne measurement with a contrast of 96% and an electronic clearance of 15dB. The fidelity of this Q function to the target state Fock state $|1\rangle$ is 31% ± 5%, which is insufficient to certify that the state is of stellar rank one or higher.

The failure to certify that our state is of stellar rank one highlights a problem in the photon subtraction stage of our experiment. From the experimental point of view this is precisely what these verification techniques are supposed to do. This shows that the DHD is a highly useful tool to find and ultimately fix experimental problems.

To further investigate this current limitation, we performed a process tomography of the photon subtractor. The matrix that describes this tomography is shown in figure 6, which represents the probability to subtract a photon in HG_0 when the gate is shaped in different HG modes. The Schmidt number of the process is 1.35, meaning that the photon subtraction is not precisely mode-selective and the subtraction process involves other modes rather than the HG_0 mode. By improving the non-linear optics to increase the mode selectivity, we aim to be able to use the DHD to create and verify a non-Gaussian state of stellar rank at least one.

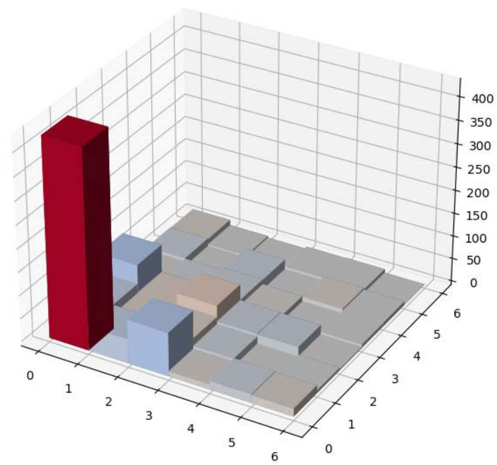


FIGURE 11 MATRIX DESCRIBING THE PROCESS TOMOGRAPHY FOR A PHOTON SUBTRACTION IN MODE HG0

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[2] A. Mari and J. Eisert, *"Positive Wigner functions render classical simulation of quantum computation efficient"*, PRL 109, 230503 (2012).

[3] U. Chabaud and M. Walschaers, *"Resources for bosonic quantum computational advantage",* PRL 130, 090602 (2023).