[DATE]

Efficient verification of quantum computing architectures with bosons

D1.1 – SINGLE-MODE VERIFICATION

Version 1.0 – Final
PU

This project has received funding from the European Union's Horizon Europe Framework Programme under Grant Agreement No.101114899

Single-mode verification

Acronym	**Veriqub**
Project Name:	**Efficient verification of quantum computing architectures with bosons**
Grant Agreement No:	**101114899**
Start Date:	**01/09/2023**
End Date:	**31/08/2027**
Contributing WP	**WP1**
WP Leader:	**INRIA (Ulysse Chabaud)**
Deliverable identifier	**1.1**
Contractual Delivery Date: 08/2025	**Actual Delivery Date: 08/2025**
Nature: Report	**Version: 1.0 Final**
Dissemination level	PU

Revision History

Version	Created/Modifier	Comments
0.0	18/07/25 Giulia Petrarulo (Project Manager)	Template creation
0.1	18/07/25 Ulysse Chabaud (Scientific coordinator)	First version with scientific content
0.2	30/07/25 Eduardo Alberto Bardales España (internal reviewer)	Reviewed the document
1.0	26/08/25 Ulysse Chabaud (Scientific coordinator)	Final version

Table of contents

1. CV VERIFICATION PROTOCOLS..5

2. OPTIMISED VERIFICATION PROTOCOLS 7

3. REFERENCES 10

Acronyms

CV	Continuous Variable

Publishable Summary

This report outlines theoretical methods for single-mode CV verification. These theoretical methods involve deriving optimised homodyne and heterodyne estimators for single-mode quantum states, compatible with detectors of non-unit quantum efficiency.

These estimators build upon a large literature developed in the context of quantum tomography, repurposing theoretical tools for the task of fidelity estimation. Compared to full-fledge quantum tomography, direct fidelity estimation has the advantage of using a minimal number of measurement samples to retrieve the sought information about the state of the quantum system under study.

In the Veriqub project, these single-mode fidelity estimation methods are used as efficient subroutines in more complex multimode fidelity witnessing protocols, allowing to bypass the prohibitive cost of tomography when probing the properties of large-scale bosonic quantum systems and devices.

1. CV VERIFICATION PROTOCOLS

CV state verification with, say, heterodyne detection, consists in the following black-box scenario:

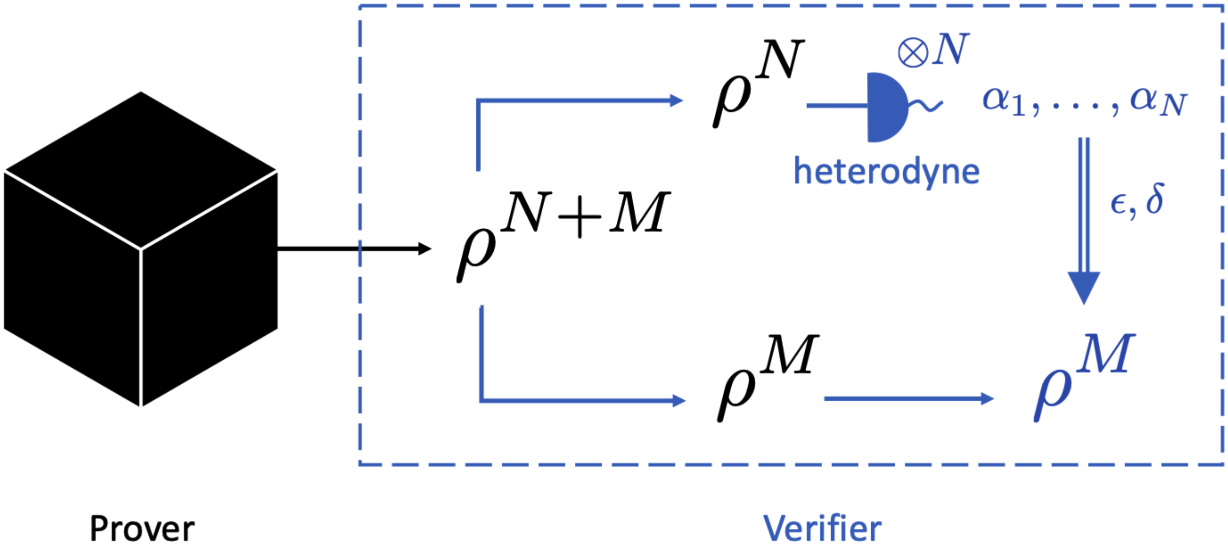


FIGURE 1. PICTORIAL DEPICTION OF A CONTINUOUS-VARIABLE HETERODYNE VERIFICATION SCHEME

A "prover" produces a quantum state to be checked by a "verifier". In the lab, the role of the prover is played by the experiment, while the experimentalist plays the role of the verifier. Here, the verifier has trusted measurements, meaning that the detection device with which they measure the tested states is fully characterized, and its behaviour can be trusted. In contrast, no assumption is made on the state generation step, hence this black-box depiction.

In the single-mode setting, the goal of a CV state verification protocol is to provide the verifier with a sequence of measurements and classical post-processing steps they must realize, at the end of which they obtain an estimate of the quantum fidelity between the state prepared by the prover and a target pure state. Specifically, in the case of heterodyne detection, the protocol consists in calculating the "distance" from the generated state to a target state via the fidelity such that

$$\mathrm{F}\left(\hat{\rho}_{\mathrm{exp}}, \Psi_{\mathrm{target}}\right) = \int \mathrm{P}_{\mathrm{target}}(\alpha)\, \mathrm{Q}_{\mathrm{exp}}(\alpha) \mathrm{d}\alpha,$$

where $\mathrm{P}_{\mathrm{target}}$ is the Glauber-Sudarshan function (or some regularisation thereof) of the target state, and $\mathrm{Q}_{\mathrm{exp}}$ is the Husimi function of the state under certification.

2. OPTIMISED VERIFICATION PROTOCOLS

In this report, we present new theoretical results on the optimised verification of single-mode states, building upon previous results by members of the Veriqub consortium [1,2]. In particular, in [1], rigorous bounds were obtained for heterodyne verification for ideal measurements.

In this context, the new results obtained regarding optimised methods for single-mode CV verification are four-fold [3]:

- Rigorous bounds for homodyne verification
- Homodyne verification compatible with detectors of non-unit efficiency.
- Heterodyne verification compatible with detectors of non-unit efficiency.
- A detailed comparison between homodyne and heterodyne verification, highlighting advantages and drawbacks of each method.

Homodyne detection of a state ρ is a single-mode Gaussian measurement that corresponds to projection on a rotated quadrature operator, which mathematically amounts to sample from a distribution $\mathrm{P}_\rho(\mathrm{x},\theta)$. Experimentally, homodyne detection is implemented by mixing an incoming signal beam with a strong local oscillator beam at the same frequency, typically using a beamsplitter. The mixed beams then interfere, and the resulting signal is measured by photodetectors. The phase and amplitude of the signal beam are inferred by adjusting the phase of the local oscillator and analyzing the difference in photodetector outputs.

On the other hand, heterodyne detection of a state ρ is a single-mode Gaussian measurement that corresponds to projection on a coherent state, which mathematically amounts to sample from the Husimi distribution $\mathrm{Q}_\rho(\alpha)$. Experimentally, heterodyne detection is implemented by splitting an incoming signal beam using a beamsplitter, and measuring both output arms with homodyne detection, with a $\pi/2$ phase shift on one of the arms. The outcomes proving the real and imaginary parts of the heterodyne measurement outcome α.

We now describe a homodyne protocol to estimate the fidelity between an experimental state and a single-mode target state with a finite support over the Fock basis.

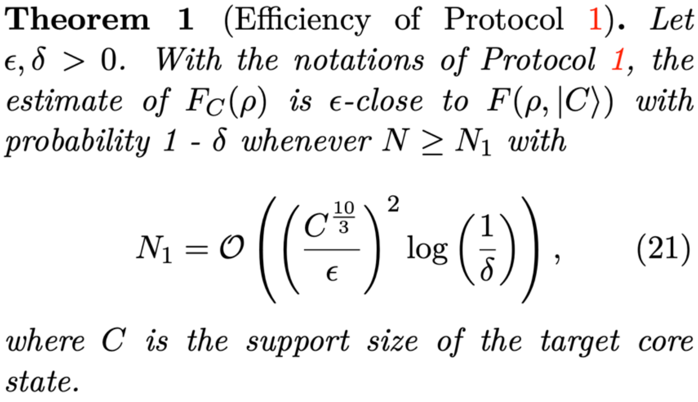
We introduce homodyne estimator functions as $\mathrm{f}_{\mathrm{lk}}(\mathrm{x})\mathrm{e}^{\mathrm{i}(\mathrm{l}-\mathrm{k})\theta}$ (for detailed expressions see [3, Section 5.2]). Averaging those estimator functions over samples from homodyne detection allows to estimate density matrix elements. Our main result is an explicit verification protocol with rigorous bounds when using these estimators for state reconstruction:

Protocol 1: Single-mode fidelity estimation with homodyne detection

Let $|\mathrm{C}\rangle = \sum_{n=0}^{c-1} c_n |n\rangle$ be a target core state (a state of finite support over the Fock basis). Let $\rho^{\otimes(\mathrm{N}+1)}$ be N+1 copies of an unknown single-mode (mixed) quantum state. Fix also a fidelity threshold for acceptance F, a precision parameter ϵ and a failure probability δ.

1. Measure N copies of ρ with homodyne detection at a random phase θ, obtaining the samples $x_1,\dots,x_N;\theta_1,\dots,\theta_N$.
2. Compute the mean F_C of the estimator over these samples.
3. If the estimated fidelity is above the threshold $F+\epsilon$, accept the remaining copy – otherwise reject.

The following result summarizes the efficiency of this protocol (here O stands for the asymptotic Landau notation):



Theorem 1 (Efficiency of Protocol 1). *Let $\epsilon, \delta > 0$. With the notations of Protocol 1, the estimate of $F_C(\rho)$ is ϵ-close to $F(\rho, |C\rangle)$ with probability 1 - δ whenever $N \geq N_1$ with*

$$N_1 = \mathcal{O}\left(\left(\frac{C^{\frac{10}{3}}}{\epsilon}\right)^2 \log\left(\frac{1}{\delta}\right)\right), \qquad (21)$$

where C is the support size of the target core state.

We refer to [3] for the proof. Since N_1 scales polynomially in $1/\epsilon$ and logarithmically in $1/\delta$, the protocol is efficient for fidelity estimation between an unknown single-mode quantum state and a core state. Moreover, since core states form a dense subset (for the trace norm) of the set of normalized single-mode pure quantum states, given any target normalized single-mode pure state, we can use our fidelity estimation protocol by targeting instead a truncation of the state in the Fock basis in order to estimate the fidelity of any single-mode continuous-variable quantum state with that state using homodyne detection.

We also extend these results to cover the case of non-unit quantum efficiency [3, Section 5.4]. This achieved by appropriately modifying the estimators to include a quantum efficiency parameter.

A similar approach, although much more technical, allows us to extend these ideas for heterodyne verification. We refer to [3, Section 6.5 and Appendix L] for a full exposition.

We conclude this report with a comparison between our new-found homodyne and heterodyne verification techniques, in the context of single-mode verification (see also [3] for an in-depth discussion).

Based on our analytical results, we find that homodyne verification is more efficient than heterodyne verification in the single-mode setting: we have unbiased estimators and quadratic polynomial dependence on the precision ε for the homodyne protocol compared to biased estimators and a higher polynomial dependence for the heterodyne protocol.

On the other hand, in the case of realistic detectors with non-unit quantum efficiency, homodyne verification requires a quantum efficiency higher than $1/2$, while heterodyne verification does not suffer from this drawback. In the context of Veriqub, we do not expect this feature to play a role, as standard quantum efficiencies of detectors are above $1/2$.

Finally, our study identifies that undoing quantum operations in classical post-processing is only possible for orthogonal passive linear unitaries in homodyne sampling, as compared to the possibility of classical post-processing all passive linear unitaries with heterodyne sampling. Therefore, heterodyne sampling may be applicable to a wider range of efficient verification scenarios as compared to homodyne sampling when looking at multimode verification. As such, they will both be pursued in the rest of the project. This analysis paves the way for the multimode verification protocols which will be demonstrated experimentally within Veriqub.

3. REFERENCES

[1] Ulysse Chabaud, Tom Douce, Frédéric Grosshans, Elham Kashefi and Damian Markham, *Building trust for continuous-variable quantum states*, Proceedings of the 15th Conference on the Theory of Quantum Computation, Communication and Cryptography (2020).

[2] Ulysse Chabaud, Frédéric Grosshans, Elham Kashefi and Damian Markham, *Efficient verification of Boson Sampling*, Quantum 5, 578 (2021).

[3] Varun Upreti and Ulysse Chabaud, *An efficient quantum state verification framework and its application to bosonic systems*, arXiv:2411.04688 (2024) – submitted to *Quantum*.