




30 APRIL 2026	Efficient verification of quantum computing architectures with bosons
	D1.1 – SINGLE-MODE VERIFICATION Version 2.0 – Final PU
	This project has received funding from the European Union's Horizon Europe Framework Programme under Grant Agreement No.101114899

Disclaimer- "The content of this publication is the sole responsibility of the TRIPLE consortium and can in no way be taken to reflect the views of the European Commission. The European Commission is not responsible for any use that may be made of the information it contains."

This deliverable is licensed under a Creative Commons Attribution 4.0 International License



Deliverable Name

Acronym	VeriQuB
Project Name:	Efficient verification of quantum computing architectures with bosons
Grant Agreement No:	101114899
Start Date:	01/09/2023
End Date:	31/08/2027
Contributing WP	WP1
WP Leader:	INRIA (Ulysse Chabaud)
Deliverable identifier	1.1
Delivery Date:	Actual Delivery Date: 04/2026
Nature: Report	Version: 2.0 Final
Dissemination level	PU

Revision History

Version	Created/Modifier	Comments
0.0	18/07/25 Giulia Petrarulo (Project Manager)	Template creation
0.1	18/07/25 Ulysse Chabaud (Scientific coordinator)	First version with scientific content
0.2	30/07/25 Eduardo Alberto Bardales España (internal reviewer)	Reviewed the document
1.0	26/08/25 Ulysse Chabaud (Scientific coordinator)	Submitted version
2.0	22/04/26 Ulysse Chabaud (Scientific coordinator)	Revised version at the end of RP2

Table of contents

PUBLISHABLE SUMMARY.....	3
1 CV VERIFICATION PROTOCOLS.....	4
2 OPTIMISED VERIFICATION PROTOCOLS	6
3 CONCLUSION	8
4 REFERENCES	8

Acronyms

CV	Continuous Variable
----	---------------------

PUBLISHABLE SUMMARY

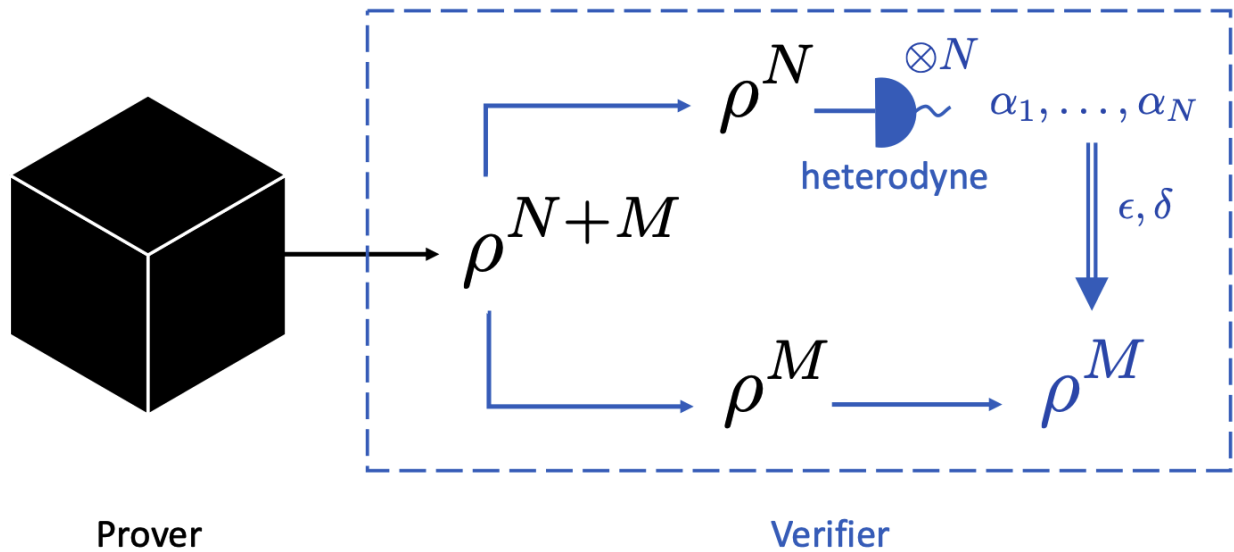
This report outlines theoretical methods for single-mode CV verification. These theoretical methods involve deriving optimised homodyne and heterodyne estimators for single-mode quantum states, compatible with detectors of non-unit quantum efficiency.

These estimators build upon a large literature developed in the context of quantum tomography, repurposing theoretical tools for the task of fidelity estimation. Compared to full-fledge quantum tomography, direct fidelity estimation has the advantage of using a minimal number of measurement samples to retrieve the sought information about the state of the quantum system under study.

In the VeriQuB project, these single-mode fidelity estimation methods are used as efficient subroutines in more complex multimode fidelity witnessing protocols, allowing to bypass the prohibitive cost of tomography when probing the properties of large-scale bosonic quantum systems and devices.

1 | CV VERIFICATION PROTOCOLS

CV state verification with, say, heterodyne detection, consists in the following black-box scenario:



A “prover” produces a quantum state to be checked by a “verifier”. In the lab, the role of the prover is played by the experiment, while the experimentalist plays the role of the verifier. Here, the verifier has trusted measurements, meaning that the detection device with which they measure the tested states is fully characterized, and its behaviour can be trusted. In contrast, no assumption is made on the state generation step, hence this black-box depiction.

In the single-mode setting, the main goal of a CV state verification protocol is to provide the verifier with a sequence of measurements and classical post-processing steps they must realize, at the end of which they obtain an estimate of the quantum fidelity between the state prepared by the prover and a target pure state. In VeriQub, the measurements considered are both homodyne and heterodyne detection.

Homodyne detection of a state ρ is a single-mode Gaussian measurement that mathematically corresponds to projection on a rotated quadrature operator, which mathematically amounts to sample from a distribution $P_\rho(\mathbf{x}, \theta)$. Experimentally, homodyne detection is implemented by mixing an incoming signal beam with a strong local oscillator beam at the same frequency, typically using a beamsplitter. The mixed beams then interfere, and the resulting signal is measured by photodetectors. The phase and amplitude of the signal beam are inferred by adjusting the phase of the local oscillator and analysing the difference in photodetector outputs.

On the other hand, heterodyne detection of a state ρ is a single-mode Gaussian measurement that corresponds to projection on a coherent state, which mathematically amounts to sample from the Husimi distribution $Q_\rho(\alpha)$. Experimentally, heterodyne detection is implemented by splitting an

incoming signal beam using a beamsplitter, and measuring both output arms with homodyne detection, with a $\pi/2$ phase shift on one of the arms. The outcomes proving the real and imaginary parts of the heterodyne measurement outcome α .

Specifically, in the case of heterodyne detection, the single-mode verification protocol consists in estimating the “distance” from the generated state to a target state via the fidelity such that

$$F(\hat{\rho}_{\text{exp}}, \Psi_{\text{target}}) = \int P_{\text{target}}(\alpha) Q_{\text{exp}}(\alpha) d\alpha,$$

where P_{target} is the Glauber-Sudarshan function (or some regularisation thereof) of the target state, and Q_{exp} is the Husimi function of the state under certification.

Given a desired confidence interval (ϵ, δ) , where $\epsilon > 0$ is a precision parameter and $\delta > 0$ is a confidence parameter, the output of the verification protocol is an estimate of the fidelity which is ϵ -precise with probability $1 - \delta$, given enough measurement samples. The theoretical challenge is to provide rigorous bounds on the number of samples sufficient to guarantee a given confidence interval when estimating the fidelity.

2 | OPTIMISED VERIFICATION PROTOCOLS

In this report, we present new theoretical results on the optimised verification of single-mode states, building upon previous results by members of the VeriQuB consortium [1,2], that were limited to ideal heterodyne measurements. In particular, in [1], rigorous confidence intervals (as a function of the number of samples) were obtained for heterodyne verification for ideal measurements.

In this context, the new results obtained are four-fold [3]:

- Rigorous confidence intervals for homodyne verification
- Homodyne verification compatible with detectors of non-unit efficiency.
- Heterodyne verification compatible with detectors of non-unit efficiency.
- A detailed comparison between homodyne and heterodyne verification, highlighting advantages and drawbacks of each method.

Concretely, the single-mode verification protocols developed in VeriQuB and reported here are optimised with respect to previously existing ones in that they are applicable to a much larger class of measurements, namely: noiseless homodyne detection (where rigorous confidence intervals were missing), noisy homodyne detection (where rigorous confidence intervals were missing), and noisy heterodyne detection (where both analytical estimator functions and rigorous confidence intervals were missing). Hereafter, we detail these new contributions.

We first describe a protocol to estimate the fidelity between an experimental state and a single-mode target state with a finite support over the Fock basis. We introduce homodyne estimator functions as $f_{\text{Ik}}(\mathbf{x})e^{i(1-k)\theta}$ (for detailed expressions see [3, Section 5.2]). Averaging those estimator functions over samples from homodyne detection allows to estimate density matrix elements. Our main result is an explicit verification protocol with rigorous bounds when using these estimators for state reconstruction:

Protocol 1: Single-mode fidelity estimation with homodyne detection

Let $|C\rangle = \sum_{n=0}^{c-1} c_n |n\rangle$ be a core state, of finite support over the Fock basis. Let $\rho^{\otimes(N+1)}$ be $N+1$ copies of an unknown single-mode (mixed) quantum state.

1. Measure N copies of ρ with homodyne detection at a random phase θ , obtaining the samples $x_1, \dots, x_N; \theta_1, \dots, \theta_N$.
2. Compute the mean F_C of the estimator over these samples.
3. If the estimated fidelity is above the threshold $F + \epsilon$, accept the remaining copy – otherwise reject.

The following result summarizes the efficiency of this protocol (here \mathcal{O} stands for the asymptotic Landau notation):

Theorem 1 (Efficiency of Protocol 1). *Let $\epsilon, \delta > 0$. With the notations of Protocol 1, the estimate of $F_C(\rho)$ is ϵ -close to $F(\rho, |C\rangle)$ with probability $1 - \delta$ whenever $N \geq N_1$ with*

$$N_1 = \mathcal{O} \left(\left(\frac{C^{\frac{10}{3}}}{\epsilon} \right)^2 \log \left(\frac{1}{\delta} \right) \right), \quad (21)$$

where C is the support size of the target core state.

We refer to [3] for the proof. Since N_1 scales polynomially in $1/\epsilon$ and logarithmically in $1/\delta$, the protocol is efficient for fidelity estimation between an unknown single-mode quantum state and a core state. Since core states form a dense subset (for the trace norm) of the set of normalized single-mode pure quantum states, given any target normalized single-mode pure state, we can use our fidelity estimation protocol by targeting instead a truncation of the state in the Fock basis in order to estimate the fidelity of any single-mode continuous-variable quantum state with that state using homodyne detection.

We also extend these results to cover the case of non-unit quantum efficiency [3, Section 5.4]. This is achieved by appropriately modifying the estimators to include a quantum efficiency parameter.

A similar approach, although much more technical, allows us to extend these ideas for heterodyne verification. We refer to [3, Section 6.5 and Appendix L] for a full exposition.

We conclude this report with a comparison between our new-found homodyne and heterodyne verification techniques, in the context of single-mode verification (see [3] for an in-depth discussion).

Based on our analytical results, we find that homodyne verification is more efficient than heterodyne verification in the single-mode setting: we have unbiased estimators and quadratic polynomial dependence on the precision ϵ for the homodyne protocol compared to biased estimators and a higher polynomial dependence for the heterodyne protocol.

On the other hand, in the case of realistic detectors with non-unit quantum efficiency, homodyne verification requires a quantum efficiency higher than $\frac{1}{2}$, while heterodyne verification does not suffer from this drawback. In the context of VeriQub, we do not expect this feature to play a role, as standard quantum efficiencies of detectors are above $\frac{1}{2}$.

3 | CONCLUSION

In the wider context of the VeriQuB project, these optimized single-mode verification protocols, applicable to both (noisy) homodyne and heterodyne detection, will be used as subroutines in more complex verification protocols. These include resource-witnessing protocols developed in WP2, as well as multimode state verification protocols further developed in WP1. In particular, our study identifies that undoing quantum operations in classical post-processing is only possible for orthogonal passive linear unitaries in homodyne sampling, as compared to the possibility of classical post-processing all passive linear unitaries with heterodyne sampling. Therefore, heterodyne sampling may be applicable to a wider range of efficient verification scenarios as compared to homodyne sampling when considering multimode verification. As such, they will both be pursued in the rest of the project. This analysis paves the way for the multimode verification protocols which will be demonstrated within VeriQuB.

4 | REFERENCES

- [1] Ulysse Chabaud, Tom Douce, Frédéric Grosshans, Elham Kashefi and Damian Markham, [*Building trust for continuous-variable quantum states*](#), Proceedings of the 15th Conference on the Theory of Quantum Computation, Communication and Cryptography (2020).
- [2] Ulysse Chabaud, Frédéric Grosshans, Elham Kashefi and Damian Markham, [*Efficient verification of Boson Sampling*](#), Quantum 5, 578 (2021).
- [3] Varun Upreti and Ulysse Chabaud, [*An efficient quantum state verification framework and its application to bosonic systems*](#), arXiv:2411.04688 (2024), submitted to Quantum.